GSBA 603: HW 4

Due Wednesday, September 22nd, 2010

1. Let $M_X(t)$ be the moment generating function of X, and define $S(t)=log(M_X(t))$. Show

$$\frac{d}{dt}S(t)\Big|_{t=0} = EX, \quad \frac{d^2}{dt^2}S(t)\Big|_{t=0} = Var\,X.$$

2. Does a distribution exist for which $M_X(t) = \frac{t}{1-t}$, $|t| < 1$? If yes find it. If no prove it.

3. For a random variable X, $M_X(t) = \frac{1}{81}(e^t + 2)^4$. Find $P(X \leq 2)$.

4. While rolling a balanced die successively, the first *6* occurred on the third roll. What is the expected number of rolls until the first *1*?

5. Let X and Y be independent positive gamma random variables with (r_1,λ) and (r_2,λ), respectively. Define $U=X+Y$ and $V=X/(X+Y)$.
   a. Find the joint pdf of U and V.
   b. Prove that U and V are independent.
   c. Show that U is gamma and V beta.

   Hint: relationship between gamma and beta functions: $B(n,m) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)}$.

6. Let X and Y be independent positive gamma random variables with common pdf:

$$f_X(x) = e^{-x} I_{\{x \geq 1\}},$$

where $I_{\{x\geq 1\}} = \begin{cases} 1, & x \geq 1 \\ 0, & x < 1 \end{cases}$. Calculate the joint pdf of $U=X/Y$ and $V=XY$.

7. 3.52

8. An insurance policy is written to cover a loss X, where X has density function

$$f_X(x) = \frac{3}{8}x^2 I_{\{0 \leq x \leq 2\}}.$$

The time (in hours) to process a claim of size x, where $0 \leq x \leq 2$, is uniformly distributed on the interval from x to $2x$. Calculate the probability that a randomly selected claim on this policy is processed in three hours or more.

9. Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \frac{1}{3} I_{\{x=0\}} + \frac{2}{3} I_{\{x=1\}}$$

Let $Y = X_1X_2X_3$. Determine the mgf, $M_Y(t)$.